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Research Article

Numerical Investigation On Thermal Conduction in Fractal-Like Porous Media

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Abstract

The paper presents two heat-conduction models in fractal-like porous media with matrix and pore space as the two fractal sources based on fractal theory. A comprehensive numerical study on heat transfer in the porous media is conducted by self-complied program based on the finite volume method. The two bi-dimensional simulation models are verified by the experimental data, which achieved a good agreement with each other. The calculated results indicate that the effective thermal conductivity decreases sharply for porosity that is smaller than a certain value and then gradually flattens to a stable value. At the same time, the connectivity of the matrix distribution and the size of the matrix channel in the direction of heat flow play decisive roles in the effective thermal conductivity, heat flow and temperature distribution of the porous media.

Keywords: Effective thermal conductivity; Fractal-like porous media; Fractal theory; Heat conduction; Numerical simulation

Introduction

The process of heat and mass transfer in porous media was very complicated and affected by many factors such as the structure of porous media. A most commonly used approach for describing the heat and mass transfer in porous media was the physical approach presented by Philip and De Vries [1-3]. The approach was the most widely used because those formulations resemble most closely the practical situations. An alternative model was given by Eckert, et al. [4,5]. Baggio et, al. [6] developed the new coupled differential equations for heat and mass in porous media. A clear classification is made of the various types of equations used and of their physical meaning. Some researchers proposed to solve the original enthalpy balance equation without invoking any further simplification for the two-phase region, while assuming the saturation enthalpies and the latent heat to be functions of saturation pressure [7-9], thereby allowing explicit heat diffusion to occur solely due to the temperature gradient even in the two-phase region. However, the main problems of the aforementioned approaches were that the specific geometric parameters of porous media could not be characterized and described in detail. Researchers mostly used the concept of "volume average" to regard porous media as periodic, uniformly distributed and virtually continuum on the macro-scale, ignoring the effects caused by the local differences and irregularities of internal structure. There was a large difference between the hypothesis and the real situation.

Therefore, investigations of heat and mass transfer in porous media for the analysis of the transmission, strain, and other properties have become focused in part on obtaining an appropriate method or model to characterize the structure of porous media. Since the 1980s, many experiments and studies have proved that the pore structure of porous media has fractal characteristic within a certain scale. Fractal geometry, with the ability to describe irregular and disordered objects that traditional Euclidean geometry fails to analyze, was formally proposed by Benoit B. Mandelbrot in 1975 to explore the complexity of objects. Accordingly, there had been a growing use of the fractal approach to characterize these chaotic, geometrically irregular physical systems. Fractal geometry has been applied to such diverse fields as material science, meteorology, ecology, urban landscapes, economics and finance, soil sciences, and medical imaging.

Fractal (covering an infinite range of scales) and prefractal (covering a finite range of scales) geometric models have been applied by many researchers to soils [10] and rocks [11-13] and show substantial potential for contributing to an improved understanding of flow and transport in porous media [14,15]. Huai et al. generated several types of fractals to model the structures of porous media and to reproduce heat conduction by the lattice Boltzmann method [16,17] and finite volume method [18].

Mathematical regular fractals, such as Sierpinski carpets, Menger sponges, and Koch curves, are often used to simulate and characterize complex structures of porous media. Fractal structures such as the Sierpinski carpet and Menger sponge are used to simulate the characteristics of particle size distribution of actual 3D complex porous media [19,20]. A fractal dimension is determined by measuring the length of a structure on a given interval of scales. Their study confirmed that higher Dn values were associated with the lower aggregate stabilities. They also pointed out that fractal dimension of soil aggregates (Dn) can be a useful index for characterizing and describing soil erodibility. The Koch curve was applied to analyze random walk paths / fluid flow lines in porous media. Wheatcraft and Tyler [21] applied simple scaling arguments to characterize the tortuous streamlines through heterogeneous media by fractal geometry. Xu, et al. [22] used fractal theory and the Monte Carlo simulation technique to develop a probability model for radial flow in fractured porous media. Analytical expressions for relative permeabilities of wetting and non-wetting phases are presented by Xu, et al. [23].

To present an investigation of the temperature distribution of porous media. This paper presents two heat-conduction models in fractal-like porous media with matrix and pore space as the two fractal sources based on fractal theory. A comprehensive numerical study on heat transfer the porous media is conducted by self-complied program based on the finite volume method. It is assumed in the model that the heat conduction is steady. In addition, the effects of porosity, matrix distribution, and pore distribution on the effective thermal conductivity and temperature distribution are analyzed.

Establishment and Simulation of Fractal Model

Following the procedure described in the literatures [21,24], two heat-conduction models in fractal-like porous media with matrix and pore space as the respective fractal sources are constructed, i.e., the matrix-fractal model and pore-fractal model. Furthermore, a bi-dimensional elliptical heat transfer calculation program is developed for this mathematical model by using FORTRAN language, Fortran PowerStation 4.0 is selected for debugging platform. The matrix and pore-fractal models are based on two different grids of 64×64 and 125×125, respectively to investigate the differences variation and regularity of local heat transfer in two conditions' porous media and simulate the heat conduction laws under different model and fractal dimension conditions. As shown in (Figure 1), there are four Sierpinski random carpet models with two grid numbers generated by two different generation methods. D_{a1}, D_{a2}, D_{a3}, and D_{a4} are used to represent the fractal dimension of these four models, respectively, and can be calculated as [25]

$$D_{a1} = D_{a2} = ln13 / ln4 \cong 1.850$$

$$D_{a3} = D_{a4} = ln20 / ln5 \cong 1.861$$

This paper simulates the porous media and carries out the plane discretization by applying the "artificial structured fractal" method to develop a random Sierpinski carpet model. In this model, each discrete control volume grid represents only one kind of material, the white grids represent the pore, and the black grids represent the matrix, simplifying the investigation of the model into a two-dimensional thermal diffusion problem of a two-elements composite material.

The thermal physical parameters of porous media are mainly affected by the internal structure and are not closely related to the heat transfer process [26]. Therefore, the heat conduction of porous media is simplified in the steady-state. And the governing equation of temperature and thermal conductivity can be written as [27]

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) = 0 \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) = 0 \qquad (1)$$

Where T is the temperature, λ is the thermal conductivity.

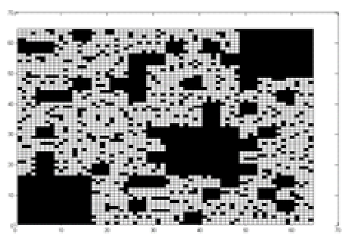
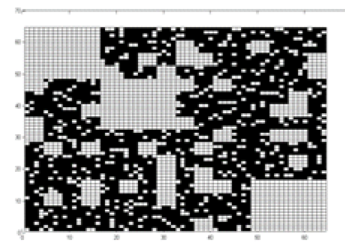
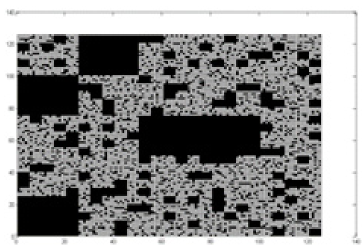
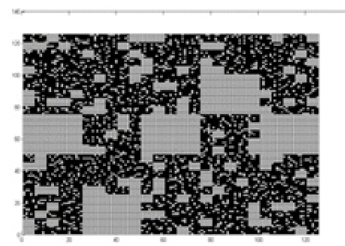
Grid numbers	Matrix as fractal source	Pore as fractal source
64×64	 (a1)	 (a2)
125×125	 (a3)	 (a4)

Figure 1: Matrix fractal model and pore fractal model with the grid numbers are 64×64 and 125×125, respectively.

The thermal conductivity for certain boundary conditions is simulated by the finite volume method (FVM). Based on the previously reported method [16], the temperature field of computational domain can be obtained by solving Equation (1). Since the heat conduction is in the steady state, the heat flux along the direction passing through any vertical cross-section is constant. By calculating the heat flux on any sections, the effective thermal conductivity of different structures with different porosity can be obtained by [16]

$$\lambda_e = \frac{Q \cdot L}{|T_2 - T_1| \cdot A} \qquad (2)$$

where λ_e is the effective thermal conductivity, Q is the heat flux at any cross -section, A is the area through which the heat is transferred, and L is the total length along the heat conduction direction. The cross-sectional height of the heat flow is assumed to be 1, and then, A is equal to L, and Equation (2) can be simplified to the following

$$\lambda_e = \frac{Q \cdot L}{|T_2 - T_1|} \qquad (3)$$

Numerical Solution

General Remark

The afore mentioned equations are discretized by the finite volume method on a staggered grid system [28,29]. The QUICK scheme and central difference scheme are used to discretize the convection term and the diffusion term respectively, and an efficient segregated algorithm, SIMPLER, is applied to the pressure-velocity solution. The algebraic equations for the whole computational domain can be solved by TDMA (Tridiagonal matrix algorithm) +ADI (Alternating direction implicit) method. The linearization method is used to deal with the source terms of the equations above. To ensure the convergence of iteration, under-relaxation of the dependent variables and pressure can be incorporated into the solution process of the algebraic equations. According to the upwards treating processes, a 2-D program has been developed to solve the model.

Converge Criteria

Convergence criterion of the iteration solution procedure can be stated as follow

$$\left| \frac{\lambda_e^{q+200} - \lambda_e^{q}}{\lambda_e^{q+200}} \right| < 10^{-6} \qquad (4)$$

Self-Complied Program Examination

The accuracy and feasibility of self-complied program was verified through experimental data. A long-term experimental test was carried out by H. Liu [30] from April 25th 2014 to August 27th 2014. In order to clearly compare the measured data with simulated results concluded from the self-complied program, a whole day's experimental data of June 8, 2014 was adopted to verify the model. The error analysis of model estimation was shown in (Figure 2). It is can see that the mean square error (MSE) is 0.57 ℃ and the mean relative error (MRE) is 6.4%. The error can satisfy the needs of the engineering practice. According to the comparison and error analysis of testing and simulated data, the self-complied program use in this paper can reliably and precisely deal with the 2D coupled heat transfer in soil.

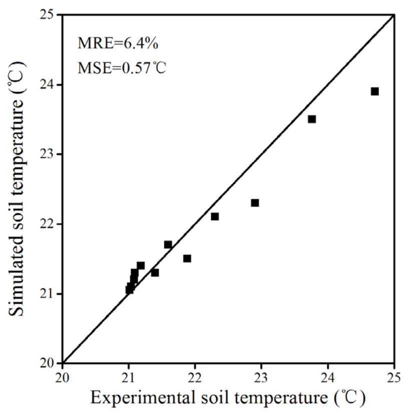


Figure 2: Comparison of the soil temperature.

Calculation Results and Analysis

The results calculated according to the method presented in this paper are shown in (Figure 3). In the logarithmic coordinate system, the effective thermal conductivity and porosity approximately satisfy a linear relationship without large deviations. The corresponding function relationship is given by

$$\lambda_{\varepsilon} \propto \varepsilon^{m} \qquad (5)$$

Where m is the slope of the line, and ε is the porosity, and the relationship follows Archie's law [31], demonstrating that the calculation method and computation program are reliable.

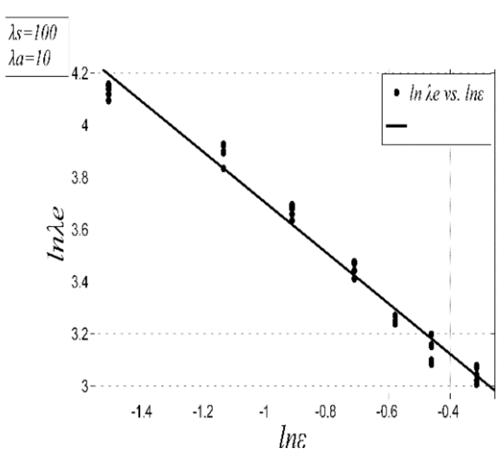


Figure 3: Effect of porosity on effective thermal conductivity.

Influence of Porosity on Effective Thermal Conductivity

For the four types of fractals in (Figure 1), suppose that the thermal conductivity of the matrix grid is equal to 100.0 and the thermal conductivity of the pore grid is equal to 10.0. The effective thermal conductivity can be calculated for different types of fractals. Due to the randomness in the generation of these structures, the calculated effective thermal conductivities are different for different runs of the computation program, and a series of data points are calculated to obtain smooth curves and the intuitive correspondence.

(Figures 4,5) show the plots of effective thermal conductivity versus porosity for the matrix and pore-fractal model, respectively. Here, the dotted line represents calculation results of the pore-fractal model, and the solid line represents the calculation results of the matrix-fractal model. As seen from the figures, both the effective thermal conductivity of the matrix-fractal model and the pore-fractal model show the same variation with the porosity. Obviously, the effective thermal conductivity decreases with increasing porosity, owing to the decrease in the components of the matrix resulting in the decrease in the thermal conductivity. In the figures, the thermal conductivity for the interval between 0 and 0.55 show almost linear scaling with the porosity, which means that the thermal conductivity is significantly affected by the porosity. When the porosity reaches a certain value of 0.55~0.6, the effective thermal conductivity no longer changes obviously with the porosity, and gradually converges to a stable value, and the thermal conductivity of the porous media also stabilizes to a very low value. The sharp change of the thermal conductivity before and after a certain porosity value analogous to the percolation threshold and referred to as the thermal conductivity threshold, representing the critical point of the transition of the porous media thermal conductivity.

In this work, the thermal conductivity threshold is used to characterize the critical point at which the porosity affects the thermal conduction state. When the medium is in the thermal conduction state, the thermal conductivity follows Fourier's law. Near the thermal conductivity threshold, the heat-conducting group has a statistical self-similar structure and can be regarded as a fractal. In this case, the effective thermal conductivity can be obtained by Fourier's law.

In the meantime, it can be seen that the variation curve of the effective thermal conductivity of the pore-fractal model is located above the matrix-fractal model, showing that the thermal conductivity of the pore-fractal model is obviously higher than that of the matrix-fractal model. By contrast, according to the conventional understanding, the matrix components should have a more positive role in promoting the thermal conductivity, in disagreement with the obtained result. Therefore, the matrix and pore fractal structure models are compared for approximately the same porosity.

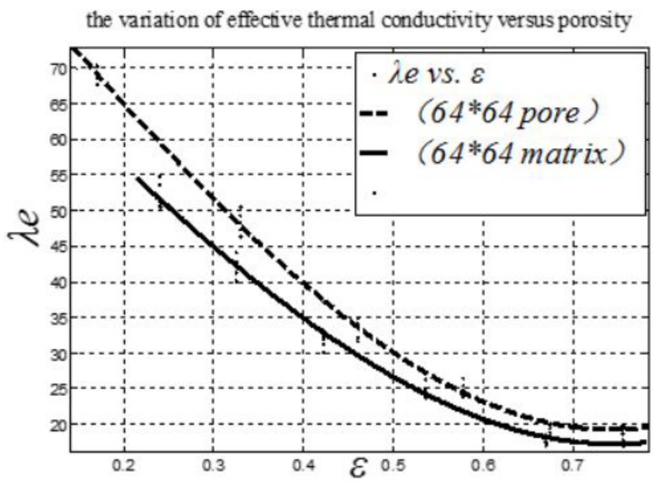


Figure 4: The variation of effective thermal conductivity versus porosity of different types of fractals with the grid.

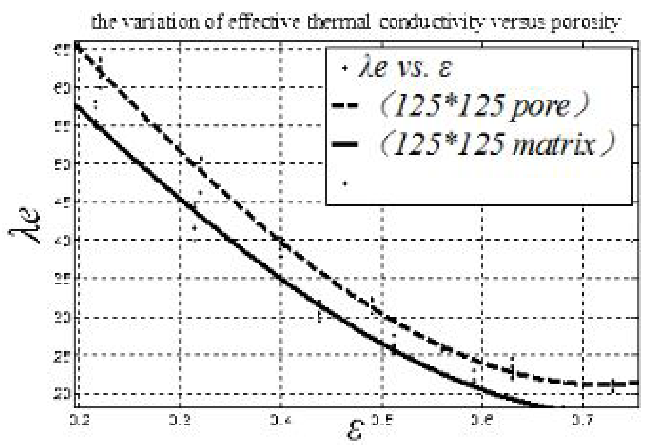


Figure 5 The variation of effective thermal conductivity versus porosity of different types of fractals with the grid numbers are 125×125.

In this work, the pore-fractal model with the porosity of ε=0.33 and the matrix-fractal model with the porosity of ε=0.325 are selected for comparison; the effective thermal conductivities of the two models are 46.28 and 39.15, respectively. It can be seen from (Figures 6,7) that the matrix distribution of the matrix-fractal model is more concentrated and that the number of the large-sized matrix is more, while the pore-fractal model is more dispersed and the matrix uniformity is higher. Then, the paper makes an analogy between thermal conduction in porous media and current transmission the porous media structure is simplified as matrix micro channels and pore micro channels, and the heat conduction of porous media can be regarded as the process of heat flow passing through these micro channels. Owing to the small conductive resistance of the matrix micro channels, the heat flow passes through them preferentially, depending mainly on the permeability of the matrix. Consequently, the matrix-fractal model characterized by fewer matrix microchannel shows poor thermal conductivity.

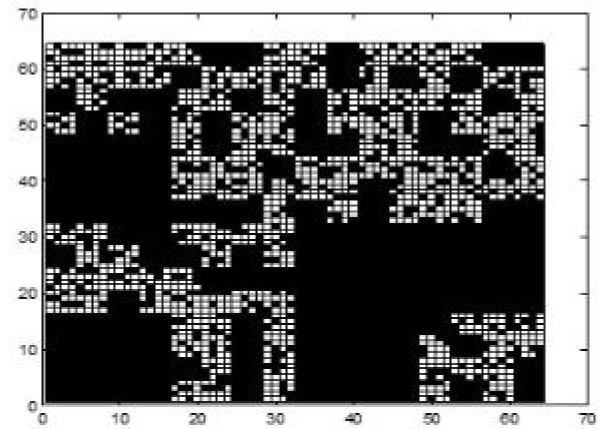


Figure 6 : Matrix fractal model of ε=0.325, λ_{ε}=39.15.

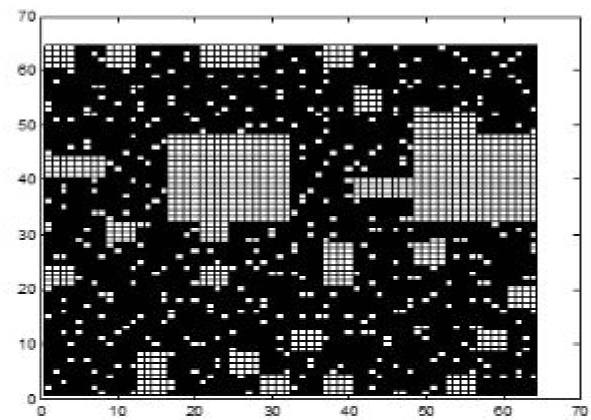


Figure 7: Pore fractal model of ε=0.33, λ_{ε}=46.28.

To verify the correctness of this argument, suppose that the thermal conductivity of the matrix grid is equals to 100.0 and the thermal conductivity of the pore grid is equals to 0.001 (insulation material). In this case, thermal conduction in porous media can be approximated as the thermal conduction of the matrix part. The calculated results indicate that the effective thermal conductivity of the matrix-fractal model is 0.008, and that of the pore fractal model is 29.26. The comparison of the two fractal models under the approximately same porosity conditions confirms the result that the connectivity of the matrix distribution promotes the thermal conductivity.

(Figures 8,9) show the variation of effective thermal conductivity versus porosity of the fractal structure objects with two different meshes of 64×64 and 125×125, respectively. It can be seen from the figures that the curves of the two models are in good agreement with the rest area except for slight differences when the porosity is below 0.25 or above 0.55, demonstrating that the number and size of the matrix and pore regions are not the dominant factors for the thermal conductivity. Rather, the connectivity of porous media matrix micro channels and the connectivity of pore micro channels are keys.

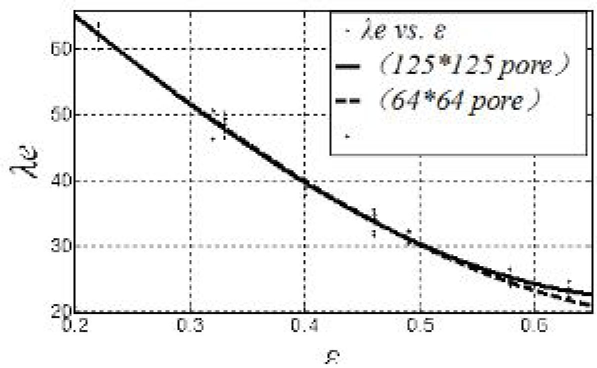


Figure 8: Comparison of the effective thermal conductivity curves of the pore fractal models with the grid numbers are 64×64 and 125×125.

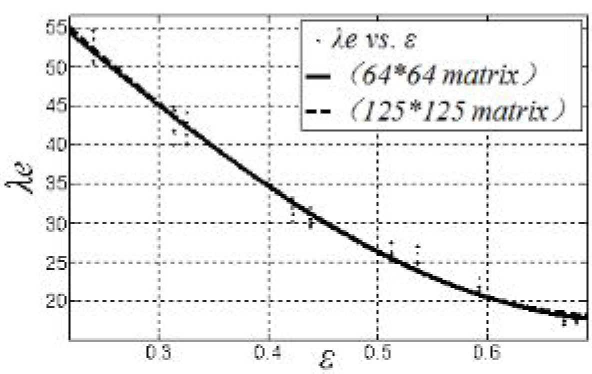


Figure 9: Comparison of the effective thermal conductivity curves of the matrix fractal models with the grid numbers are 64×64 and 125×125.

In addition, it can be seen from the figures that the effective thermal conductivity values at each porosity are not fixed, but rather are found in a certain range. It is clear that the fluctuation of the effective thermal conductivity is caused by different structures of the porous media model. Specifically, the fluctuation is caused by the pore size, matrix size and spatial distribution. Therefore, the paper compared porous media models, temperature distributions, and heat flux distributions to analyze the specific conditions.

(Figure 10) shows the variation of effective thermal conductivity at different porosity of the matrix-fractal model and pore-fractal model, respectively. Porous media models with 10 different fractal structures were obtained based on random Sierpinski carpets. Each line in (Figure 10) represents the calculation of effective thermal conductivity with the same porosity but different structure, and different lines represent different porosity. As seen from (Figure 10), the structural differences between the models have do not significantly influence on the thermal conductivity for large porosity. Compared to the pore-fractal models, the effective thermal conductivity curves of the matrix-fractal models show a large difference, indicating that the influence of the matrix distribution on porous media thermal conductivity is very strong. Therefore, this paper focuses on the analysis of the influence of the matrix distribution.

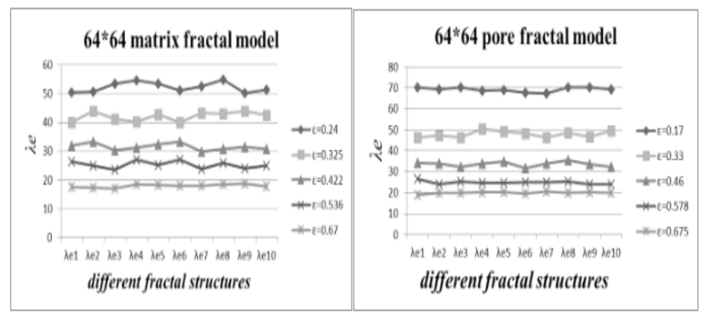


Figure 10: Effective thermal conductivity of different structures and different porosity.

The Effect of Pore and Matrix Distribution On Heat Flux and Temperature Field Distribution

In this paper, the matrix-fractal models corresponding to the two extreme values of effective thermal conductivity are selected. As shown in (Figure 11), the heat flow direction is from left to right, and the matrix distribution forms two belts, perpendicular to the heat flow and parallel to heat flow, respectively. The calculated results show that the effective thermal conductivity of the matrix distribution belt in the direction perpendicular to the heat flow is 38.12 (the minimum value), less than that of the other fractal models with arbitrary distributions. This lower value is because the pore distribution in the direction of the heat flow is similar to the formation of a “wall” that resists the heat flow to a certain extent. The effective thermal conductivity of the matrix distribution belt in the direction parallel to the heat flow is equals to 48.76 (the maximum value). The matrix distribution belt is likely to form a wide heat passage, resulting in a very large enhancement of the heat flow. Therefore, the connectivity of the matrix channel in the heat flow direction is the key factor affecting the thermal conductivity of the porous media. Higher thermal conductivity is obtained for a greater number of channels and larger channel size of the channel in the direction of the heat flow.

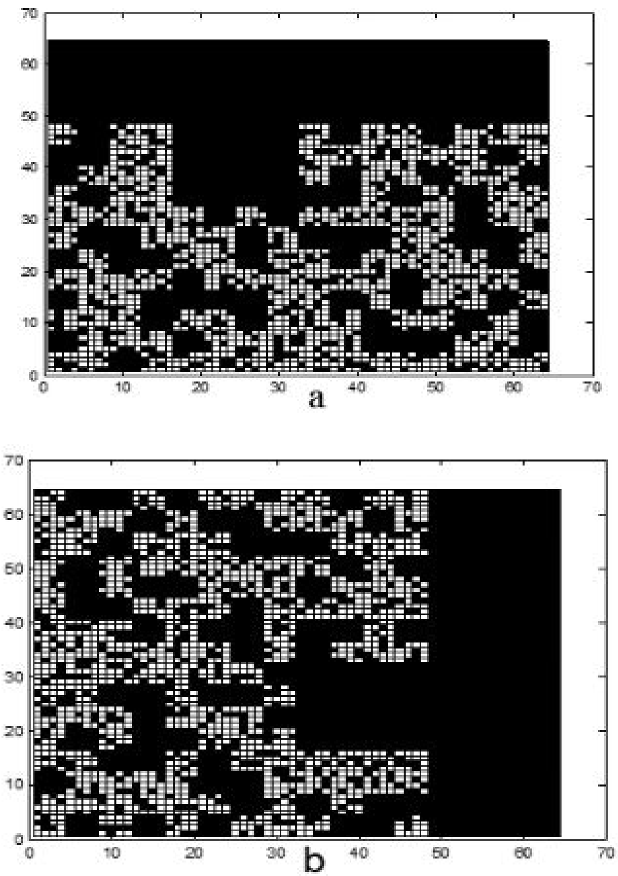


Figure 11: Matrix fractal models. For the matrix distribution belt in the direction of the heat flow as shown in (Figure 11(a)) For the matrix distribution belt whose direction is perpendicular to the heat flow as shown in (Figure 11(b)).

(Figure 12) shows the temperature contour map and the corresponding heat flow distribution of the pore-fractal model. In the figures, the circle area is the large pore area of the model. The figures show that for the macro-pores, the temperature gradient and the temperature contours are close. At the non-uniform distribution of the matrix and the pores, the temperature contours are distributed more evenly and sparsely. It is clear from the corresponding heat flux distribution shown in (Figure 12(b)), that the intensity of the heat flux varies greatly, with obvious peaks and troughs. In the macro-pores area, large temperature gradient and the small but relatively uniform heat-flow intensity result from the small thermal conductivity of the pores, corresponding to the troughs in (Figure 12(b)).

In the nonuniform region, the heat flow intensity is very high, and the position of the crest is often in the narrow substratum between the pores, verifying that the heat flow passes through the matrix micro channels preferentially, and that the heat flux intensity is inversely related to the microchannel size.

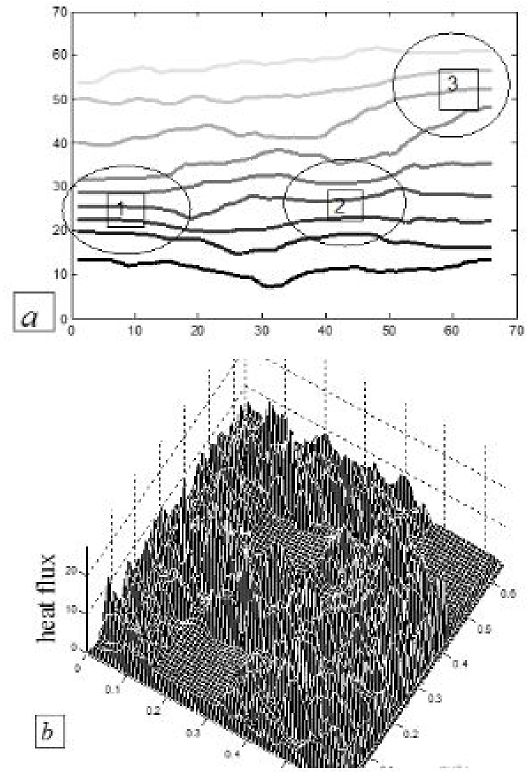


Figure 12: Temperature contour map **(a)** and heat flow distribution diagram **(b)** of heat conduction model.

Conclusion

Based on fractal theory, and considering the randomness of the structure, two different fractal thermal conductivity models, namely, the matrix-fractal model and pore-fractal model, were constructed based on the Sierpinski carpet.

The calculated results indicate that the effective thermal conductivity shows a certain change with the porosity of porous media, and the effective thermal conductivity of the pore fractal model is significantly higher than that of the matrix fractal model. Comparison of the two fractal models under the same porosity confirmed that the connectivity of the matrix distribution promotes the thermal conductivity. Suppose that the thermal conductivity of the matrix grid is equals to 100.0 and that the thermal conductivity of the pore grid is equals to 0.001 (insulation material), showing that numerical simulations verify the correctness of the relationship between the connectivity of the matrix distribution and the thermal conductivity.

Selecting two special matrix fractal models to quantitative analysis, it is shown that the size of the matrix channel in the direction of heat flow is the key factor affecting the thermal conductivity of porous media.

Visualization of the temperature distribution and heat flow distribution, intuitively demonstrates the specific effects of the internal structure of the porous media on the heat conduction process.

Remarks

The existing research work still has some deficiencies. Therefore, further study is needed for more precise and more reasonable model that can reduce simulation errors.

The lack of methods for simulations of the moisture migration of porous media based on the fractal model and for simulations of the heat transfer problem under the coupling of heat and moisture migration hinders the application of fractal theory to specific problems. Owing to the randomness of the 2D/3D fractal model, the connectivity of the pores affects the migration of the moisture, which in turn affects heat conduction, and the entire field of heat and humidity. At the same time, for the solution of the heat-moisture coupling nonlinear equation established on the fractal model, as well as the setting of the moisture content, permeability and the moisture diffusion coefficient of the pore mesh points in the model, the differences in the local structure will lead to the local variations in the corresponding values.

Currently, the fractal theory is used to study the heat and mass transfer in porous media and pore adsorption phenomena [32]. Most of these methods directly simplify the porous media to a fractal model. However, the actual porous media in actual conditions may not be well-represented by a fractal structure. In addition to using the fractal dimension and the percolation threshold as evaluation criteria, it is also necessary to derive a specific rule for determining when these criteria are valid.

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